

# Math 122 / Problem Set 1

Written problems due Monday, September 26

*Monday, September 19*

1. A square matrix  $A$  is called *nilpotent* if  $A^k = 0$  for some  $k > 0$ . Prove that if  $A$  is nilpotent, then  $I + A$  is invertible.

2. We define the *trace* of a square matrix to be the sum of its diagonal entries:

$$\operatorname{tr} A = a_{11} + a_{22} + \cdots + a_{nn}.$$

(a) Show that  $\operatorname{tr}(A + B) = \operatorname{tr} A + \operatorname{tr} B$ .

(b) Show that  $\operatorname{tr} AB = \operatorname{tr} BA$ .

(c) Show that the equation  $AB - BA = I$  has no solutions in  $n \times n$  matrices with real entries.

**Reading:** Artin §2.1

*Wednesday, September 21*

3. Let  $G$  be a group, with composition law  $\cdot$ . Define  $G^\circ$  to be equal to  $G$  as a set, and equip it with the composition law  $a \circ b = b \cdot a$ . Show that  $G^\circ$  is a group.

**Reading:** Artin §2.2

*Friday, September 23*

4. Prove that in any group the orders of  $ab$  and  $ba$  are equal.

5. Prove that a group in which every element except the identity has order 2 is abelian.

**Reading:** Artin §2.3, 2.4